

RECOVERING THE MEANING OF “CRITIQUE” IN CRITICAL MATHEMATICS EDUCATION

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The different social arenas in which mathematics is involved pose important challenges for mathematics education that cannot be understood independently of the multiple social functions that it is expected to fulfil (Skovsmose, 1994; D’Ambrosio, 2002; Valero, 2004). If the purpose of education is to allow societal development in a democratic way, then it is not enough for people to learn a “petrified” mathematics, isolated from its social implications in the world. The recognition that mathematics is present in many of our daily activities and decisions has been pushing the mathematics education community to rethink the aims of school mathematics (Frankenstein, 1983; Skovsmose, 1994; Gutstein, 2003). Since most of the mathematics “ruling” our world is hidden, students need to critically deconstruct the way in which mathematics formats reality, so that they can socially participate as informed and critical citizens in the construction of a democratic, socially just society. This has been the call of critical mathematics education.

To our knowledge, Frankenstein (1983) was the first person to use the term “critical” in connection with mathematics education. The conceptualization of “critique” in the work of Frankenstein comes from Freire’s liberatory pedagogy, where the notions of *conscientização* (critical consciousness) and *transformation* are crucial to thinking about educational practice (Freire, 1998). Against this background, and mainly working in adult education, Frankenstein developed the “critical mathematical literacy” program. This approach sought to increase the mathematical confidence of adult students through a collaborative approach where political and social issues are directly related with the learning of mathematics. Students are asked not just to solve some particular mathematical problem, but a mathematical problem potentially coming from all areas of everyday life—political, economic and social—as a way to critically analyse how mathematics is used to manipulate people’s decisions and how they can use it to interpret information, make informed decisions and transform their (often oppressed) realities.

Ten years after Frankenstein coined the term “critical mathematics education,” Skovsmose published his book *Towards a philosophy of critical mathematics education*, in which he launched the philosophical sparks for a critical mathematics education utilising the critical theory developed by the Frankfurt School. Skovsmose (1994) understands critical education as one that addresses the conflicts and cri-

sis in society by uncovering inequalities and oppression of whatever kind (p. 22). Addressing the critical role played by mathematics in society implies an understanding of the risks and uncertainties that mathematics and societal progress conveys. In the field of mathematics education, a critical approach can involve confronting students with situations in which mathematics seems to format the way they understand and act upon reality.

Within the theoretical frameworks informing Skovsmose’s and Frankenstein’s work, there is, despite the differences, [1] a strong affinity. The presence of Freire’s theory in Skovsmose’s notion of *mathemacy*, as well as the notion of “dialogic learning and teaching” (Alrø & Skovsmose, 2002), and the fact that Freire himself was informed by theories coming from the Frankfurt School of critical theory, are probably the most visible qualities. Together, Frankenstein and Skovsmose, can arguably be considered the authors of the seminal and most influential work within critical mathematics education.

These perspectives have spread their influence through mathematics education, to the point of being adopted by teachers who felt the need to change their practices. This is true of Ana, a Portuguese mathematics teacher and one of the authors of this article. We explore Ana’s attempt to bring into her classroom topics of critical mathematics education. Particularly, we highlight the problems Ana encountered in her practice when trying to implement topics of critical mathematics education in school. These problems can be considered as “symptoms” of the failure of critical mathematics education. Instead of considering the problems faced by Ana as something that can be “healed”—through better teaching practices, for instance—we posit them as a window into the entire contradiction of schooling.

Where do we stand?

The problems faced by the mathematics education community cannot be fully grasped within the strictly didactical perspective that animates the majority of the research being done in the field (Pais, 2012; Pais, Stenoft & Valero, 2010). The closure of the field around the categories of “learning” and “mathematics” has inhibited mathematics education research from achieving a broader comprehension of the role school mathematics has in society (Pais & Valero, in press). In order to understand the dynamics of the teaching and learning of mathematics and the way research results

influence what is happening in mathematics classrooms, we need to contextualize these practices within the social modes of living that characterize the world today. This kind of analysis requires looking at research from a socio-political perspective (Valero, 2004) that explicitly aims to connect the role of research—in particular in mathematics education—to the discourses and ideologies that fuel our current society.

As a way of understanding the role of politics in the so-called “micro-context” of schools and mathematical learning, we find support in the work of Slavoj Žižek, who, in the last two decades, has been actively engaged in recovering the outdated notion of *ideology* as a crucial concept with which to understand the dynamics of our current capitalist society. Particularly important for our argument will be the notion of *symptom*. Despite being originally a clinical or psychoanalytical notion, Lacan (2007) noticed how, in historical terms, it was Karl Marx who invented it. If we are addressing a person, then a symptom is some disruption that perturbs the homeostasis of his or her physical or mental state. In Lacanian psychoanalysis, the proper way to address a symptom is not to try to “heal” it, that is, to erase its trace from the body of the patient, but to “make it talk.” In other words, instead of getting rid of these malfunctions in order to keep the healthy Ego, the psychoanalytical method seeks to suspend the patient’s Ego in order to confront the patient with his or her “dirty water”, that is, the dysfunctions which inhibit him or her from achieving a psychic equilibrium. Thus, it is the symptom and the process of its interpretation, which allows us to grasp the functioning of the person’s entire psychic system.

Why, then, was it Marx who invented the symptom? In *The Ethics of Psychoanalysis*, Lacan (2008) describes the ideology of *evolutionism* as implying a belief in a Supreme Good, in a final goal of evolution which guides its course from the very beginning. This perspective is notably evident in the influential works of John Rawls and Jürgen Habermas. Despite their differences, they share the assumption that a theory of the Social should be primarily concerned with the delineation of a set of universal principles that should guide our action towards a better society. According to Mouffe (2005), who has developed a powerful critique of such approaches to democracy, Rawls and Habermas do not deny that there will be obstacles to the realization of the ideal discourse, but those obstacles are conceived as *empirical* ones. That is, they are not seen as being intrinsic to the conceptualization of democracy involved in the evolutionist thesis, but as practical and empirical limitations of social life that, with time and effort, will naturally be surpassed.

What, in this thesis, are seen as subsidiary problems of a “good” system are, in Marxian theory, the points at which the “truth”, the immanent antagonistic character of the system, erupts (Žižek, 1989, p. 114). Capitalism and its ideology posit progress, equality and freedom as natural ideals shared by all humankind. These ideals are presented as the goals we have to strive for—we know what we want, so the question is how to achieve it. The fact that we are today (still) living in an unequal society is seen by today’s liberal-democracy as simple variation, a degeneration of the normal functioning of society that can be abolished

through improvement of the system. Through the deployment of Hegelian dialectics, Marx showed not only that the Ideal cannot be achieved in a capitalist system, but that the ideal discourse itself functions as the proper staged discourse (*i.e.*, ideology) that makes capitalist reality sustainable and acceptable. We need to know that the goal for which we all strive is equality and freedom (that the presupposition of the system is a “good” one), so that we can accept the unequal reality in which we live.

In this way, by inventing the symptom, Marx called our attention to the fact that such empirical obstacles are the necessary conditions for the maintenance of the system which generates them, and that it is through them that we can perceive the antagonistic structure of society. Our challenge in this article will be to conceive the everyday problems that a teacher lives in her work when trying to implement critical mathematics education in a regular school, not as temporary, correctable glitches, but as core points where it is possible to see a crucial and often disavowed purpose of the school system: sorting people by means of (school) credit accumulation. [2] For this purpose, we take the standpoint that a critical methodological approach in research in education has not just to do with the way the researcher engages with the participants, but also the way the researcher makes sense of the empirical reality addressed. Reality is seen as contradictory, full of curves and spins, and a critical methodology is one that tries to find a language to express these contradictions in a way that does not neglect them, nor clean the research from them, but takes them as part of the core focus.

In order to enlighten this tension between a research that cleans reality of contradictions and a critical one, we will discuss the work of one of us, Ana, a mathematics teacher in a Portuguese secondary school and Master’s graduate. During her Master’s study, Ana was confronted with several difficulties while trying to implement critical mathematics education in her mathematics class. She decided, however, not to mention these difficulties in her final dissertation (Alves, 2007), concluding that despite all the constraints she felt, it was possible and fruitful to bring critical mathematics education into the mathematics classroom. We see the difficulties faced by Ana not as marginalities, as things to be avoided or small details of a school system, but as core problems of the current school system and of society that interrupt what could be a radical emancipatory mathematics education. Therefore, in an analogous way to the psychoanalytical interpretation discussed above, we take the difficulties and constraints felt by Ana not as particular problems to get rid of, but as central issues for educational research.

A study in critical mathematics education

In the first part of this section, Ana describes her work to develop and implement critical mathematics education tasks in her teaching. In the second part, we collectively discuss some of the difficulties she faced.

My interest in critical mathematics education is partly the result of a concern with the way mathematics is traditionally taught in schools: as something disconnected from students’ reality. I see my role as a mathematics

teacher as an important factor in allowing my students to become participative, active, competent, critical citizens. The ways I have found to accomplish this aim are diverse, one being the development of activities with students in which they can uncover and understand the role of mathematics in different social situations. I adopted Skovsmose's idea of mathemacy, as the competence to analyse and reflect upon the mathematics behind a world strongly structured around mathematical modelling (Skovsmose, 1994). In my Master's research, which took place during the first period of 2006 with a class of 9th graders (aged between 14 and 16 years old) I explored the implementation of critical mathematics education.

The Portuguese curriculum explicitly mentions that "mathematics education has the purpose of helping students to uncover the mathematics behind diverse situations, promoting the education of participative, critical and confident citizens" (ME-DEB, 2001, p. 58, our translation from Portuguese). This statement was one incentive for me to attempt to implement critical mathematics education in my classroom. However, I had to recontextualize the official discourse of the curriculum. [3] Such recontextualization included choosing tasks that bring to light the social dimension of mathematics, making sure that the theme of the task is relevant to students' social reality, and assuring that the task will allow a critical analysis of the mathematics behind the modelling of a specific social activity. I carried out informal conversations with my students in order to know better their concerns and social interests. I soon realised that students' concerns were strongly connected to daily life activities ranging from the use of public transportation, to eating habits and media. After this informal survey I spent time studying the best way to construct a task that brought together the development of mathematical competences and the possibility of engaging in a critical discussion about the way mathematics formats some of the students' daily activities. As a result, I prepared two tasks: "Supermarket promotions" and "A taxi trip".

"Supermarket promotions" confronted students with some of the mathematical models informing the way we participate in our current society as consumers, by exploring how students engage in shopping when going to the supermarket. On the other hand, "A taxi trip" tried to bring together important social issues such as the recent rise in the price of fuel and all the social consequences that follow. In Portugal, for instance, the rise in the price of fuel has led the population to use more public transportation, which, in principle, should be more economic. This task was intended to critically analyse the advantages (or disadvantages) of using public transportation (in this case, taxis) instead of private modes of transportation.

At this point, everything seems ready to implement the tasks. So what are the difficulties that I faced?

The first issue is Ana's decision not to implement her critical mathematics education tasks in the regular schedule of the mathematics class. She decided to invite some students to form a club outside the hours designated for mathematics, where they could work on the tasks. The reason for this decision was that 9th grade students have a final exam at the end of the year, on which their final grades depend, as well as approval to enrol in 10th grade. Here we see the contradiction between the official discourse, present in the curriculum, and classroom practice, where it is the exam that delineates the content and form of teaching. In particular, although it is good and innovative to implement such topics, there is an inner pressure to conform that teachers are aware of. This pressure pushes Ana to implement the tasks in such a way that they do not directly challenge the school system and do not change any core features of the structure of schooling. On the other hand, implementing critical mathematics education as a separate activity makes explicit the fact that critical mathematics education is not part of the curriculum and, hence, creates in pupils' minds the idea that it is, perhaps, not really mathematics.

Another aspect of Ana's research that we want to highlight is the criteria that she used to choose the students to interview. She opted for those who had shown more interest and enthusiasm within the sessions, and justified this choice by mentioning their visibility: "choosing those who appeared more involved and participative in the sessions was a way of guaranteeing the collection of data [...] I choose the students who gave more visibility to their involvement" (Alves, 2007, p. 66, our translation). This is an option that most researchers choose (finding the "best" informants), as they need to provide clear evidence of their claims. Ana, for example, wanted to highlight the potential of critical mathematics education for developing citizenship. Therefore, it was not appropriate to choose students who, in one way or another, did not engage so enthusiastically with such experiences. On the other hand, the selection of the students was also related to the aim of her research. This type of selection is a case of what Valero (2002) calls the cleaning of research—putting aside conflicts and constraints so that research is presented in a harmonious and positive way.

Finally, Ana justified the lesser involvement of some students because, on the one hand, they were not familiar with the way they could work in the club (which was more unstructured than the classroom environment) and, on the other hand, they were still attached to a vision of mathematics as a static science having nothing to do with real life situations. Although these arguments could be true, we suggest that other issues are at stake. The lesser involvement of the students could be due to the fact that they knew that these activities would not contribute directly to prepare them for the tests and to get a good mark at the end of the year. Using Vinner's (1997) description of school as a credit system, we could say that students felt that the activities would not give them much credit. [4]

Recovering the core meaning of "critique"

Ana's research highlights methodological concerns that are characteristic of a critical methodology. The most evident one is the assumption by the researcher of her subjectivity. Ana is well aware of the difficulties of implementing critical

mathematics education in schools, the resistance of the students to such topics, the pressure to fulfil the entire disciplinary program and the need to prepare students for the final exam. This is an example of what Valero (2004) calls “making the researcher visible” (p. 19), in which the products of the research process, the intentionality of the researcher, and the paths that the researcher selected are open to the critical examination of the reader.

Despite all the difficulties, Ana did assume that it is possible and desirable to develop critical mathematics education tasks with students, and that this could be a way of promoting a bigger societal transformation. That is, Ana endorsed the evolutionist thesis and saw these difficulties as contingent obstacles which, although constraining the implementation of critical mathematics education, did not compromise its global purpose. However, if the goal of critical mathematics education is not merely a “didactical” one, but above all a *societal* one, having to do with emancipation from social forms of oppression, then a closer look should be given to the way in which these obstacles can be seen not as a contingency but as a *necessity* of the same system which promotes the implementation of critical mathematics education in schools. The obstacles—what we called the symptoms—cannot be erased without questioning the school system as a whole.

In this last section, we would like to address some philosophical issues involved in our discussion, by recovering what we consider to be the core meaning of critique both in Freirean Pedagogy and the Frankfurt School. We argue that the notion of critique has suffered from a kind of *domestication* designed to keep its radical emancipatory potential at a safe distance.

We should start by clarifying how critical theory, as developed within the Frankfurt School, understood society and existing social relations. In other words, what was the core focus of the social and political critique developed by these scholars? The answer is capitalism. Despite major differences between members of the Frankfurt School in their assessment of the development of capitalism, their respective analyses were informed by Marxian tenets (Held, 1980). According to Benhabib (1994), the core feature of Critical Theory, as it emerged in the works of Max Horkheimer, Theodor Adorno, Herbert Marcuse, Leo Löwenthal, Friedrich Pollock and Walter Benjamin, was the realisation that a revolutionary transformation of capitalism from within capitalism itself was doomed to fail. Critical theory was confronted with the enterprise of thinking of a radical alternative to an entire economic system.

Although initially the critique was focused on political economy, with time it gave place to a critique of instrumental reason, as a response to a positivist paradigm which restricted research to the activity of outlining correlations between well-defined phenomena. These two critiques did not coincide; rather the critique of instrumental reason surpassed the critique on political economy:

The transformation of the critique of political economy into the critique of instrumental reason signals not only a shift in the *object* of critique, but, more significantly, in the *logic* of critique. (Benhabib, 1994, p. 79)

The work of Habermas exemplifies this shift in the logic of critique. In his work, political economy is not so much a matter of *infrastructure*, of class struggle, but a matter of administration and technique, due to a change in which politics becomes the sphere for the technical elimination of dysfunctions and the avoidance of risks threatening “the system” (Held, 1980). This split provoked a displacement of the way the political was conceived: capitalism became naturalized and accepted, and transformation started to be conceived as being within capitalism. This disavowal of Marxism in theory was accompanied by the discarding of communism as a political system. After the fall of the Berlin Wall and the dismantling of the Soviet Union, communism was doomed to be associated with a dark, totalitarian past and the end of history, to use Francis Fukuyama’s words, was proclaimed: capitalism becomes the global social, political and economic system, to which there seems to be no alternative.

Although not so accurate in Freire’s work, [5] the Marxist vein is easily visible in the way he conceptualizes the thematic of *change*. Freire was well aware that no matter how intense and spread the local struggles in which we are involved could be, without a dialectical change involving both the economy and the *superstructure* of society, these local struggles easily end up being co-opted by the dominant ideology:

Cultural action occurs at the level of superstructure. It can only be understood by what Althusser calls “the dialectic of overdetermination”. This analytic tool prevents us from falling into mechanistic explanations or, what is worse, mechanistic action. An understanding of it precludes surprise that cultural myths remain after the infrastructure is transformed, even by revolution. (Freire, 1998, p. 480)

These “mechanistic actions” that Freire mentions are nothing less than the capitalist demands for perpetual reforms by means of integrating what could be new and potentially emancipatory acts into well established social structures.

We suggest that critical mathematics education exemplifies the power of Capital in co-opting what is officially presented as an emancipatory enterprise. While at one level the discourse of critical mathematics education is one of emancipation, when actualized in schools it becomes “overdetermined” by the logic of school accreditation. Why is it possible (and even promoted—the curriculum encourages teachers to work on such topics) for Ana to introduce critical mathematics education in her teaching, but not to change the exams-based evaluation system? The message we get is that it is fine to change teaching methods, learning strategies and even the curricular content—what can be called, after Marx, the superstructure of schooling—as long as the core features of the system (capitalist schooling based on accreditation, the infrastructure) remains the same.

Indeed, mathematics education research gives an extensive array of choices for teachers willing to improve students’ learning. However, this appearance of choice should not deceive us. To paraphrase Žižek (2006, p. 348), it is the mode of appearance of its very opposite: of the absence of any real choice with regard to the fundamental

structure of school, where year after year teachers are asked to mark students with a grade that will determine (sometimes in quite severe ways) their future possibilities. The impossibility of choosing the core features that map school life must be disguised by the availability of a multitude of choices that do not change anything. Critical mathematics education can very well perform the role of what Freire (1998) calls “superficial transformations” (p. 508), designed precisely to prevent any real change in the core features of schooling.

If we recover the critique of political economy developed in the first years of the Frankfurt School, we can say that emancipation from capitalism has failed completely. In our current society, all emancipatory actions are thought and put into action within capitalism. No radical alternative to this economic system has been proposed. But this fact contrasts with the proliferation of the idea of critique, especially in education. It is in this sense that the word “critique” has become domesticated; it has lost its most radical meaning. It is a case of what Žižek (1994) calls “progressive amnesia” (p. 9): we recover critical theory but it is deprived of its true transformative core. It is fine to take a critical stance as long as you do not raise questions that could undermine the foundations of capitalist schooling. Critical mathematics education becomes possible only within the confines of school’s credit system.

As we saw, when confronted with the rigidity of the school evaluation system, Ana decided not to jeopardize this same system, developing instead her critical mathematics with a particular group of students outside the official mathematics class. At the time, Ana saw this contradiction as a difficulty, as a problem she had to overcome in order to open a space to promote critical education to her students. But what this contradiction shows is that the most important role of the teacher within the system is to prepare students for the final exam. It is good to work with students on these “radical” topics as long as they do not change the smooth functioning of schools as credit systems. Against the belief that fuels critical mathematics education that we can retain the consistency of school mathematics by getting rid of the embarrassing symptoms which disturb this consistency, we argue that such symptoms are indeed *endemic* and *necessary* to the reproduction of the system. Our proposal is that such embarrassing symptoms should be made the core focus of critical mathematics education research (instead of, for instance, research only concerned with discussing the potentialities and the positive experiences carried out under the insignia of critical mathematics education). That is, the symptoms are to be addressed not as a minor detail, to be rapidly discarded in a footnote, but as the core of a system in which the “ideal discourse” only serves to conceal the economic role of schooling.

Conclusion

In this article, we have sought to deepen the theoretical understanding of the problems felt by teachers when trying to implement powerful ideas in schools, such as the ones emanating from critical mathematics education research. We have shown how the actual circumstances of schooling—accreditation, selection—provide the concrete meaning of well-intentioned actions. We have drawn attention to the

way ideology is at work in critical mathematics education in the way potentially emancipatory actions intended to produce change (such as those carried out by Ana) get caught in a system that uses them to assert its intention to change, while its main features remain unchangeable. These main features are the fact that schools are places of social selection, and teachers are agents of exclusion, even (or especially) if they refuse to recognize it. The spirit of societal change animating critical mathematics education research cannot leave unaddressed the worldwide accreditation system; this system is indifferent to local attempts to overcome the particular problems standing in the way of a critical mathematics education.

Some will say that such an awareness of the problem takes us to an impasse. By realizing that schools are overdetermined by capitalist ideology, we are faced with the monstrous task of—if the purpose is radical emancipation—ending schools as we know them. In the current circumstances, this does not seem possible. However, what dooms us to repeated failure is precisely experiencing the change as impossible—we acknowledge that to achieve the desired emancipatory goals of critical mathematics education requires a fundamental societal change which we experience as impossible. There is no easy way out.

For our part, we adopt the old Pascalian maxim: “Be optimistic in practice, while pessimistic in theory”. In practice, we completely support the struggle of many teachers (although they are a minority—how many teachers worldwide have ever heard about critical mathematics education?) to promote activities that raise students’ critical consciousness of the role of mathematics in society. At the same time, what should also be part of this consciousness is the way school co-opts such practices so that they will, in the end, reproduce what they criticize. Thus, although in practice we should be optimistic, our role in theory is to have a broader understanding of the dynamics of change, precisely by positing mathematics and its education not merely as school subjects responsible for the acquisition of knowledge and competences by students, but also as core features of schooling’s credit system. This is the arena in which our article was written. And this is a theoretical article, in which we dare to be pessimistic about the belief that, with time, and through our local struggles we will achieve the desired change. This jump from quantity to quality is based on a “leap of faith”: there are no guarantees that such a change will occur. Indeed, by disavowing the Marxian primacy of economy—what we called the infrastructure—present in schools by means of the credit system, critical mathematics education can easily become another curricular topic that, despite the rhetoric of emancipation, only reinforces the dominant ideology. And this overdetermination needs to be understood if we really want to achieve radical emancipation.

Notes

[1] Frankenstein follows a Marxist, class centered approach, informed by Freirean pedagogy and critical pedagogy. While Skovsmose, follows a Habermasian critique of the “instrumental rationality”. More recently, Skovsmose has broadened his theoretical landscape, to include insights from Walter Benjamin and Michel Foucault. See for instance Skovsmose (2011), which offers a symptomatic background for the critique we develop in this article.

[2] As first pointed out by Vinner (1997), and further developed by Baldino (1998) and Baldino and Cabral (1998).

[3] Ana draws on official documents, such as curriculum guidelines, subjects programs and others, produced by official recontextualizing agents and recontextualizes them in a way that allows her to introduce CME in the classroom. To do that, Ana takes advantage of a set of discourses and practices, available within the field of recontextualization, and subsumes them under their aims and purposes (Morgan, Tsatsaroni & Lerman, 2002). Among such discourses are those produced by teacher training courses and educational masters and circulated within the Unofficial Pedagogic Recontextualizing Field (UPRF) (Bernstein, 1996).

[4] It is always useful to remember the research carried out by Baldino & Cabral (1998), where they show how students in school are primarily worried about passing and not necessarily about learning mathematics.

[5] Indeed, as the reader may notice in the following quotation, Freire's emphasis on the superstructure as the locus of change shows a clear disavowal of the role the primacy infrastructure has in Marxian theory.

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References

- Alrø, H. & Skovsmose, O. (2002) *Dialogue and Learning in Mathematics Education: Intention, Reflection, Critique*. Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Alves, A. (2007) *Educação Matemática Crítica na Sala de Aula* [Critical Mathematics Education in the Classroom]. Masters thesis. Lisbon, Portugal: University of Lisbon.
- Baldino, R. (1998) School and surplus-value: contribution from a third-world country. In Gates, P. (Ed.) *Proceedings of the First International Conference on Mathematics Education and Society*, pp. 73-81. Nottingham, UK: Centre for the Study of Mathematics Education.
- Baldino, R. & Cabral, T. (1998) Lacan and the school's credit system. In Olivier, A. & Newstead, K. (Eds.) *Proceedings of 22nd Conference of the International Group for the Psychology of Mathematics Education*, vol. 2, pp. 56-63. Stellenbosch, South Africa: University of Stellenbosch.
- Benhabib, S. (1994) The critique of instrumental reason. In Žižek, S. (Ed.) *Mapping Ideology*, pp. 66-92. London, UK: Verso.
- Bernstein, B. (1996) *Pedagogy, Symbolic Control and Identity: Theory, Research, Critique*. London, UK: Taylor and Francis.
- D'Ambrosio, U. (2002) *Etnomatemática: Elo entre as Tradições e a Modernidade* [Ethnomathematics: Linking Tradition with Modernity]. Belo Horizonte, Brazil: Autêntica.
- Frankenstein, M. (1983) Critical mathematics education: an application of Paulo Freire's epistemology. *Journal of Education* **165**(4), 315-339.
- Freire, P. (1998) Cultural action for freedom. *Harvard Educational Review* **68**(4), 471-521.
- Gutstein, E. (2003) Teaching and learning mathematics for social justice in an urban, Latino School. *Journal for Research in Mathematics Education* **34**(1), 37-73.
- Held, D. (1980) *Introduction to Critical Theory: Horkheimer to Habermas*. Cambridge, UK: Hutchinson & Co.
- Lacan, J. (2007) *The Other Side of Psychoanalysis: The Seminar of Jacques Lacan Book XVII*. (trans. R. Grigg). New York, NY: W.W. Norton & Company.
- Lacan, J. (2008) *The Ethics of Psychoanalysis: The Seminar of Jacques Lacan Book VII*. (Ed. J.A. Miller, trans. D. Porter). New York, NY: Routledge.
- ME-DEB (2001) *Currículo Nacional do Ensino Básico: Competências Essenciais* [National Curriculum: Essential Competences]. Ministério da Educação, Departamento do Ensino Básico.
- Morgan, C., Tsatsaroni, A. & Lerman, S. (2002) Mathematics teachers' positions and practices in discourses of assessment. *British Journal of Sociology of Education* **23**(3), 445-461.
- Mouffe, C. (2005) *The Democratic Paradox*. London, UK: Verso.
- Pais, A. (2012). A critical approach to equity in mathematics education. In Greer, B. & Skovsmose, O. (Eds.), *Opening the Cage: Critique and Politics of Mathematics Education*. Rotterdam: Sense.
- Pais, A., Stentoft, D. & Valero, P. (2010) From questions of how to questions of why in mathematics education research. In Gellert, U., Jablonka, E. & Morgan, C. (Eds.) *Proceedings of the Sixth International Mathematics Education and Society Conference*, vol. 2, pp. 398-407. Berlin: Freie Universität Berlin, MES6.
- Pais, A. & Valero, P. (In press) Researching research: mathematics education in the political. To appear in *Educational Studies in Mathematics*.
- Skovsmose, O. (1994) *Towards a Philosophy of Critical Mathematics Education*. Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Skovsmose, O. (2011) Critique, generativity and imagination. *For the Learning of Mathematics* **31**(3), 19-23.
- Valero, P. (2002) The myth of the active learner: from cognitive to socio-political interpretations of students in mathematics classrooms. In Valero, P. & Skovsmose, O. (Eds.) *Proceedings of the Third International Mathematics Education and Society Conference*, pp. 542-553. Copenhagen: Centre for Research in Learning Mathematics.
- Valero, P. (2004) Socio-political perspectives on mathematics education. In Valero, P. & Zevenbergen, R. (Eds.) *Researching the Socio-Political Dimensions of Mathematics Education: Issues of Power in Theory and Methodology*, pp. 5-24. Norwell, MA: Kluwer.
- Vinner, S. (1997) From intuition to inhibition—mathematics education and other endangered species. In Pehkonen, E. (Ed.) *Proceedings of the 21st Conference of the International Group for Psychology of Mathematics Education*, vol. 1, pp. 63-78. Lahti, Finland: PME.
- Žižek, S. (1989) *The Sublime Object of Ideology*. London, UK: Verso.
- Žižek, S. (1994) *The Metastases of Enjoyment: Six Essays on Women and Causality*. London, UK: Verso.
- Žižek, S. (2006) *The Parallax View*. Cambridge, MA: MIT Press.