Antenna arrays for pattern control

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Introduction

Main objective of COST 296:
Develop an increased knowledge of the effects imposed by the ionosphere on practical radio systems, and for the development and implementation of techniques to mitigate the deleterious effects of the ionosphere on such systems.

In Final Report of COST 284:
Geo-stationary satellites were expected to require very large multi-feed reflector antennas and very versatile arrays and array feeds.

The mitigation of the interference can be performed by imposing appropriated nulls on the radiation pattern and by controlling the sidelobe levels.

Antenna arrays play an important role in the mitigation of the ionospheric effects.
Comparing with linear arrays, **planar arrays** provides a greater control of the radiation pattern.

Circular arrays permit a fast rotation of the main beam by 360° only by shifting the excitation of the elements.

Usually, the methods for the synthesis on planar arrays are iterative or based on the sampling of continuous apertures.

The time consumed in the calculations is very high (several minutes or hours for the computation of the array excitation).
Basic Concepts

For the farfield, a spatial translation of any source only modifies the produced field in the phase:

\[ c_1(d) \rightarrow F_1(\theta, \phi) \]

\[ a c_1(d + d_0) \rightarrow a e^{j \beta_0 \cdot d} F_1(\theta, \phi) \]

Fourier Relation:

\[
F(\beta_x, \beta_y, \beta_z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(x, y, z) e^{j(\beta_x x + \beta_y y + \beta_z z)} \, dx \, dy \, dz \\
= (2\pi)^3 \mathcal{F}^{-1}[c(x, y, z)]
\]

\[
c(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(\beta_x, \beta_y, \beta_z) e^{-j(\beta_x x + \beta_y y + \beta_z z)} \, d\beta_x \, d\beta_y \, d\beta_z \\
= \frac{1}{(2\pi)^3} \mathcal{F}[F(\beta_x, \beta_y, \beta_z)]
\]
Rectangular Arrays

A planar array with elements in a rectangular grid provides two dimensions for the pattern control.

For a continuous array:

$$F(\beta_x, \beta_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c(x, y) e^{j(\beta_x x + \beta_y y)} \, dx \, dy$$

$$\beta_x = \beta \sin(\theta) \cos(\phi)$$

$$\beta_y = \beta \sin(\theta) \sin(\phi)$$

$$c(x, y) = \frac{1}{(2\pi)^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(\beta_x, \beta_y) e^{-j(\beta_x x + \beta_y y)} \, d\beta_x \, d\beta_y$$

For discrete arrays:

$$F(\beta_x, \beta_y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} c(x_n, y_m) \delta(x - x_n, y - y_m) e^{j(\beta_x x + \beta_y y)} \, dx \, dy$$

$$= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} c(x_n, y_m) e^{j(\beta_x x_n + \beta_y y_m)}$$

The fast Fourier transform can be applied.
Circular Arrays

For patterns with circular symmetry is more appropriated to use circular arrays.

The Fourier relation is applied in the same manner as presented previously. However, is more appropriated to deal with polar coordinate system,

\[
\begin{align*}
x &= \rho \cos(\phi) & \beta_x &= \xi \cos(\psi) \\
y &= \rho \cos(\phi) & \beta_y &= \xi \sin(\psi) \\
\rho^2 &= x^2 + y^2 & \xi^2 &= \beta_x^2 + \beta_y^2
\end{align*}
\]

Source distribution in polar coordinates system

\[
\begin{align*}
F(\xi, \psi) &= \int_0^\infty \int_0^{2\pi} c(\rho, \phi) e^{i[\xi \cos(\psi) \rho \cos(\phi) + \xi \sin(\psi) \rho \sin(\phi) \rho \rho \sin(\phi)]} d\rho d\phi \\
&= \int_0^\infty \int_0^{2\pi} c(\rho, \phi) e^{-i\rho \xi \cos(\psi - \phi)} \rho d\rho d\phi
\end{align*}
\]

Array factor in polar coordinates system
For radial symmetry of the source distribution and taking into account that

\[ J_0(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{jx\cos(\alpha)} d\alpha \]

the array factor is

\[
F(\xi) = \int_0^\infty c(\rho) \left[ \int_0^{2\pi} e^{j\rho \xi \cos(\varphi)} d\varphi \right] \rho d\rho
\]

\[ = 2\pi \int_0^\infty c(\rho) J_0(\rho \xi) \rho d\rho \]

The well-known result for a continuous disc of uniform current.

For **discrete arrays**, the array factor can be also given by

\[
F(\beta_x, \beta_y) = \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} c(x_n, y_m) e^{j(\beta_x x_n + \beta_y y_m)}
\]
Comparing with equispaced grids, the periodicity is destroyed in circular arrays.
Using the previous changing of variables

\[ E(\xi, \psi) = \sum_{m=0}^{M-1} \sum_{n=0}^{N_m-1} c(\rho_m, \phi_n) e^{j[\xi \cos(\psi) \cdot \rho_m \cos(\phi_n) + \xi \sin(\psi) \cdot \rho_m \sin(\phi_n)]} \]

\[ = \sum_{m=0}^{M-1} \sum_{n=0}^{N_m-1} c(\rho_m, \phi_n) e^{j\rho_m \xi \cos(\psi - \phi_n)} \]

\[ \phi_n = \frac{2\pi n}{N_m} \]

This result could be also obtained sampling a continuous source distribution.
The relation between polar and spherical variables is

\[
\xi = \sqrt{\beta_x^2 + \beta_y^2} = \sqrt{\beta^2 \sin^2(\theta) \cos^2(\phi) + \beta^2 \sin^2(\theta) \sin^2(\phi)} = \beta \sin(\theta)
\]

\[
\psi = \arctan\left(\frac{\beta_y}{\beta_x}\right) = \arctan\left(\frac{\beta \sin(\theta) \sin(\phi)}{\beta \sin(\theta) \cos(\phi)}\right) = \phi
\]

For an array distribution with radial symmetry,

\[
F(\xi) = \sum_{m=0}^{M-1} \sum_{n=0}^{N_{m-1}} c(\rho, \varphi) \sum_{k=\infty}^{\infty} j^k J_k(\rho \cdot \xi_0) e^{jk\left(\psi - \frac{2\pi}{N_m} n\right)}
\]

\[
= \sum_{m=0}^{M-1} \sum_{k=\infty}^{\infty} j^k J_k(\rho \cdot \xi_0) e^{jk\psi} \sum_{n=0}^{N_{m-1}} e^{-j \frac{2\pi}{N_m} kn}
\]

\[
= \sum_{m=0}^{M-1} \sum_{p=\infty}^{\infty} J_{N_m p}(\rho \cdot \xi_0) e^{jN_m p\left(\psi + \frac{\pi}{2}\right)}
\]

\[
e^{j \rho \cdot \xi} \cos(\psi) = \sum_{k=\infty}^{\infty} j^k J_k(\rho \cdot \xi_0) e^{jk(\psi - \varphi)}
\]

Usually, few terms of \( p \) are enough to obtain a good approximation of the arrays factor.
Fist terms of the summation:

\[ \rho = 0.5\lambda \]

\[ \rho = \lambda \]

\[ \rho = 1.5\lambda \]
Source distribution for discrete circular array,

\[
\mathcal{C}(\rho_m, \phi_n) = \frac{1}{(2\pi)^2} \int_0^\infty \int_0^{2\pi} F(\xi, \psi) e^{i\xi \cos(\psi \cdot \rho_m \cos(\phi_n) + \xi \sin(\psi \cdot \rho_m \sin(\phi_n))} \xi \, d\xi \, d\psi \\
= \frac{1}{(2\pi)^2} \int_0^\infty \int_0^{2\pi} F(\xi, \psi) e^{i\rho_m \xi \cos(\phi_n) \xi} \xi \, d\xi \, d\psi
\]

For those cases where an independence of \( \psi \) exists,

\[
\mathcal{C}(\rho_m) = \frac{1}{(2\pi)^2} \int_0^\infty F(\xi) \left[ \int_0^{2\pi} e^{i\rho_m \xi \cos(\phi_n) \xi} \, d\psi \right] \xi \, d\xi \\
= \frac{1}{2\pi} \int_0^\infty F(\xi) J_0(\rho_m \xi) \xi \, d\xi
\]

resulting in the Hankel transform of zero order.

\[
F(q) = 2\pi \int_0^\infty f(r) J_0(qr) r \, dr \\
f(r) = \frac{1}{2\pi} \int_0^\infty F(q) J_0(qr) q \, dq
\]
Results

Rectangular array:

$N=9$
$d=0.5\lambda$
81 elements

Imposing the peaks amplitude

$SLL=-13$ dB

$SLL=-30$ dB
Circular array:

\( M = 6 \)
\( N_1 = 6; N_2 = 12; N_3 = 18; N_4 = 24; N_5 = 20 \)
\( d = 0.5\lambda \) between rings
81 elements

\[ |F(\beta_x, \beta_y)| \text{ dB} / \lambda / \lambda / \lambda / \lambda / \lambda / \lambda / \lambda / \lambda \]

Uniform SLL = -18.5 dB
Since the desired array factor does not depend on $\phi$,

\[ M=6 \]
\[ N_1=6; \quad N_2=12; \quad N_3=18; \quad N_4=24; \quad N_5=20 \]
\[ d=0.5\lambda \text{ between rings} \]
\[ 81 \text{ elements} \]

\[ F(\xi) = \begin{cases} 
1 & \xi < 1.3 \\
0 & \text{otherwise}
\end{cases} \]

\[ \zeta(\rho_m) = \frac{1}{2\pi} \int_{0}^{\infty} F(\xi) J_0(\rho_m \xi) \xi d\xi \]

Current in each ring:

- $\rho_0 : 1$
- $\rho_1 : 0.95$
- $\rho_2 : 0.80$
- $\rho_3 : 0.59$
- $\rho_4 : 0.36$
- $\rho_5 : 0.15$

$SLL=-31.5 \text{ dB}$
Source distributions:

Rectangular array

Circular array

The feed system of the circular array is simpler than the one of the rectangular grid since several elements have the same excitation.
Conclusions

• The Fourier Relation method was applied to circular arrays in order to control the radiation pattern.

• Comparing with rectangular arrays, the uniform source distribution permits lower sidelobe levels.

• As it was demonstrated, lower sidelobes can be imposed only controlling the source distribution between rings. In each ring the distribution is uniform whilst for rectangular arrays usually it is necessary to control the value of all elements.